

Module 2 DC Circuit

Lesson 7

Superposition Theorem
in the context of dc
voltage and current
sources acting in a
resistive network

Objectives

- Statement of superposition theorem and its application to a resistive d.c network containing more than one source in order to find a current through a branch or to find a voltage across the branch.

L.7.1 Introduction

If the circuit has more than one independent (voltage and/or current) sources, one way to determine the value of variable (voltage across the resistance or current through a resistance) is to use nodal or mesh current methods as discussed in detailed in lessons 4 and 5. Alternative method for any linear network, to determine the effect of each independent source (whether voltage or current) to the value of variable (voltage across the resistance or current through a resistance) and then the total effects simple added. This approach is known as the superposition. In lesson-3, it has been discussed the properties of a linear circuit that satisfy (i) homogeneity property [response of output due to input $= \alpha u(t)$ equals to α times the response of output due to input $= u(t)$, $S(\alpha u(t)) = \alpha S(u(t))$ for all α ; and $u(t)$ = input to the system] (ii) additive property [that is the response of $u_1(t) + u_2(t)$ equals the sum of the response of $u_1(t)$ and the response of $u_2(t)$, $S(u_1(t) + u_2(t)) = S(u_1(t)) + S(u_2(t))$]. Both additive and multiplicative properties of a linear circuit help us to analysis a complicated network. The principle of superposition can be stated based on these two properties of linear circuits.

L.7.1.1 Statement of superposition theorem

In any linear bilateral network containing two or more independent sources (voltage or current sources or combination of voltage and current sources), the resultant current / voltage in any branch is the algebraic sum of currents / voltages caused by each independent sources acting along, with all other independent sources being replaced meanwhile by their respective internal resistances.

Superposition theorem can be explained through a simple resistive network as shown in fig.7.1 and it has two independent practical voltage sources and one practical current source.

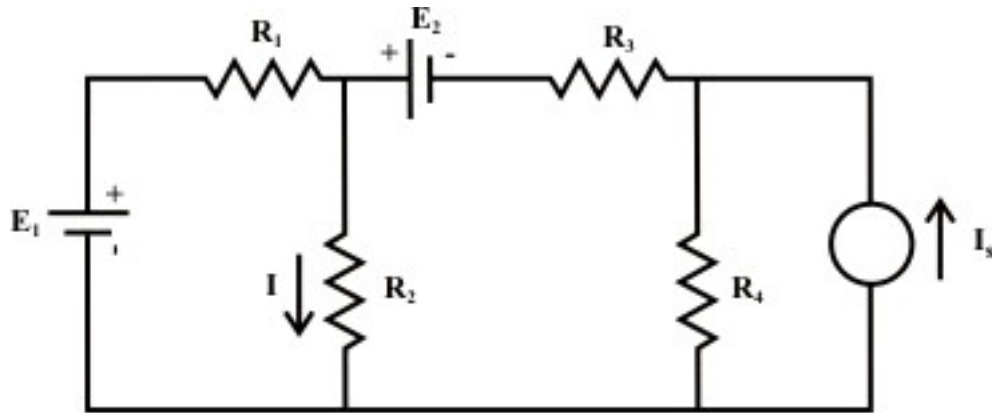


Fig. 7.1

One may consider the resistances R_1 and R_3 are the internal resistances of the voltage sources whereas the resistance R_4 is considered as internal resistance of the current source. The problem is to determine the response I in the resistor R_2 . The current I can be obtained from

$$I = I' \big|_{\text{due to } E_1 \text{ (alone)}} + I'' \big|_{\text{due to } E_2 \text{ (alone)}} + I''' \big|_{\text{due to } I_s \text{ (alone)}}$$

according to the application of the superposition theorem. It may be noted that each independent source is considered at a time while all other sources are turned off or killed. To kill a voltage source means the voltage source is replaced by its internal resistance (i.e. R_1 or R_3 ; in other words E_1 or E_2 should be replaced temporarily by a short circuit) whereas to kill a current source means to replace the current source by its internal resistance (i.e. R_4 ; in other words I_s should be replaced temporarily by an open circuit).

Remarks: Superposition theorem is most often used when it is necessary to determine the individual contribution of each source to a particular response.

L.7.1.2 Procedure for using the superposition theorem

Step-1: Retain one source at a time in the circuit and replace all other sources with their internal resistances.

Step-2: Determine the output (current or voltage) due to the single source acting alone using the techniques discussed in lessons 3 and 4.

Step-3: Repeat steps 1 and 2 for each of the other independent sources.

Step-4: Find the total contribution by adding algebraically all the contributions due to the independent sources.

L.7.2 Application of superposition theorem

Example- L.7.1 Consider the network shown in fig. 7.2(a). Calculate I_{ab} and V_{cg} using superposition theorem.

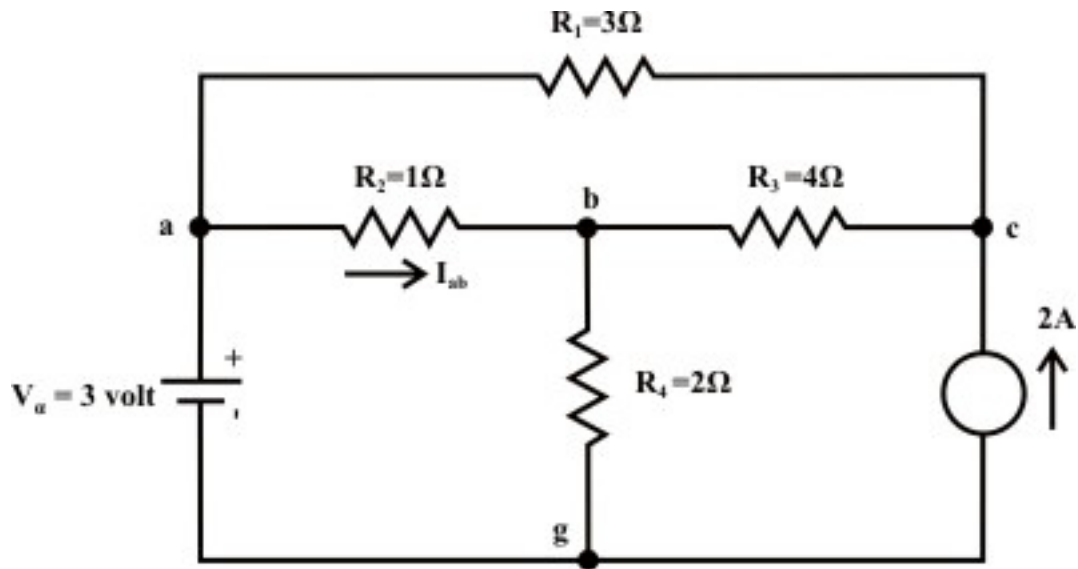


Fig. 7.2(a)

Solution: Voltage Source Only (retain one source at a time):

First consider the voltage source V_a that acts only in the circuit and the current source is replaced by its internal resistance (in this case internal resistance is infinite (∞)). The corresponding circuit diagram is shown in fig.7.2(b) and calculate the current flowing through the 'a-b' branch.

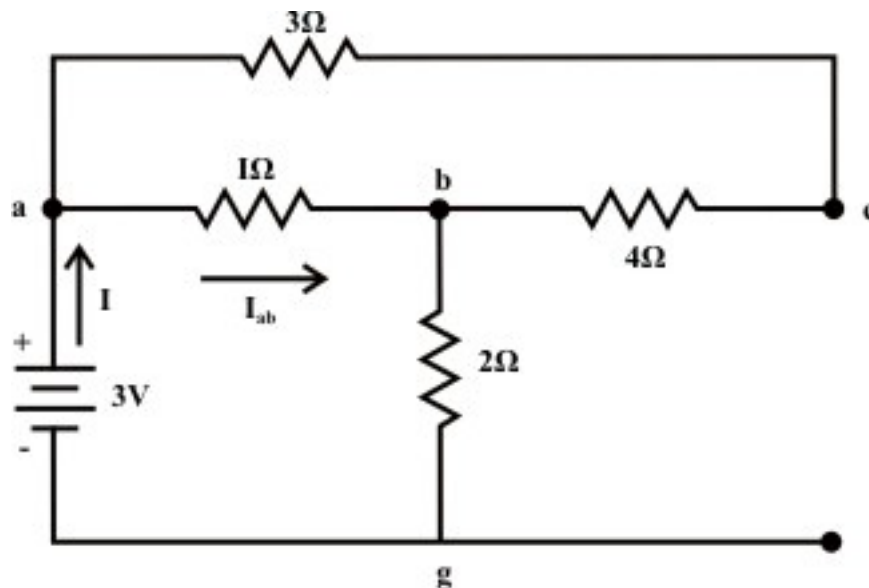


Fig. 7.2(b)

$$R_{eq} = [(R_{ac} + R_{cb}) \parallel R_{ab}] + R_{bg} = \frac{7}{8} + 2 = \frac{23}{8} \Omega$$

$$I = \frac{3}{\frac{23}{8}} A = 1.043A; \quad \text{Now current through a to b, is given by}$$

$$I_{ab} = \frac{7}{8} \times \frac{24}{23} = 0.913A \text{ (a to b)}$$

$$I_{acb} = 1.043 - 0.913 = 0.13A$$

Voltage across c-g terminal :

$V_{cg} = V_{bg} + V_{cb} = 2 \times 1.043 + 4 \times 0.13 = 2.61 \text{ volts}$ (Note: we are moving opposite to the direction of current flow and this indicates there is rise in potential). Note 'c' is higher potential than 'g'.

Current source only (retain one source at a time):

Now consider the current source $I_s = 2A$ only and the voltage source V_a is replaced by its internal resistance which is zero in the present case. The corresponding the simplified circuit diagram is shown below (see fig.7.2(c)& fig.7.2(d)).

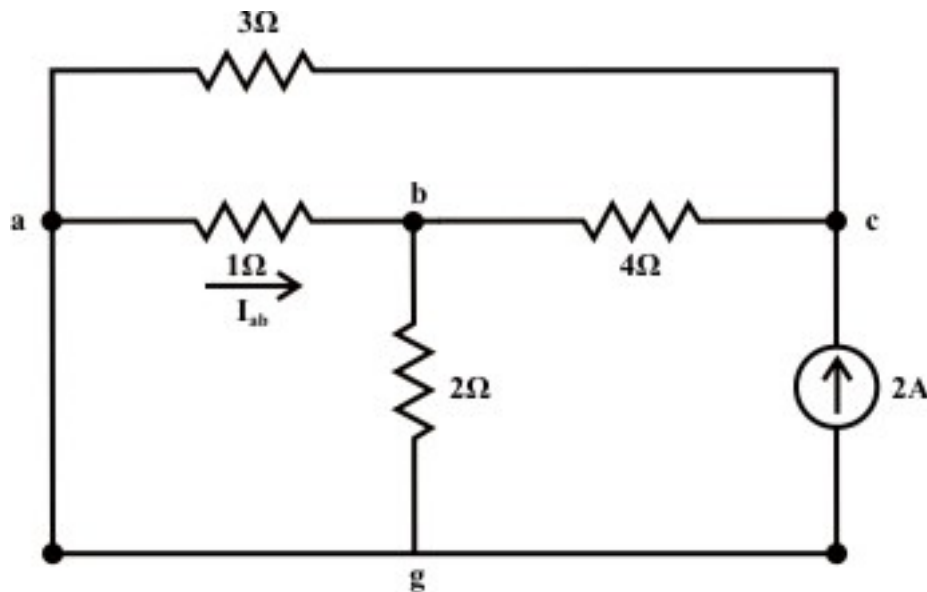


Fig. 7.2(c)

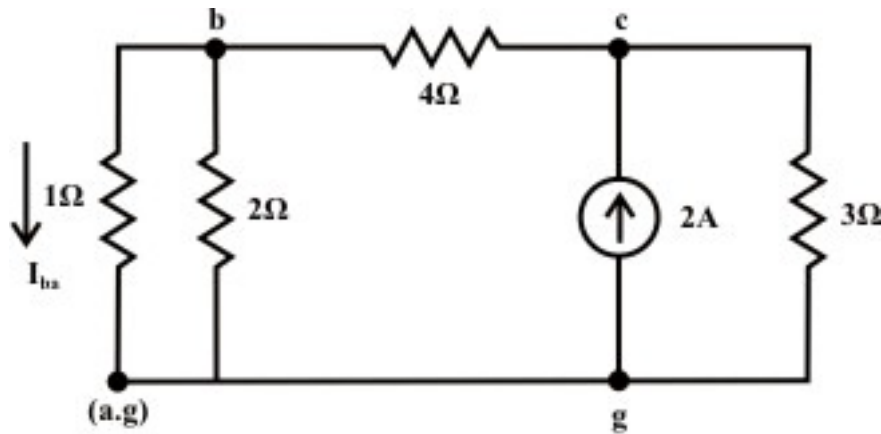


Fig. 7.2(d)

Current in the following branches:

$$3\Omega \text{ resistor} = \frac{(14/3) \times 2}{(14/3) + 3} = 1.217A; \quad 4\Omega \text{ resistor} = 2 - 1.217 = 0.783A$$

$$1\Omega \text{ resistor} = \left(\frac{2}{3}\right) \times 0.783 = 0.522A \text{ (b to a)}$$

$$\text{Voltage across } 3\Omega \text{ resistor (c \& g terminals)} V_{cg} = 1.217 \times 3 = 3.651 \text{ volts}$$

The total current flowing through 1Ω resistor (due to the both sources) from a to b = 0.913 (due to voltage source only; current flowing from 'a' to 'b') – 0.522 (due to current source only; current flowing from 'b' to 'a') = $0.391A$.

Total voltage across the current source $V_{cg} = 2.61 \text{ volt}$ (due to voltage source ; 'c' is higher potential than 'g') + 3.651 volt (due to current source only; 'c' is higher potential than 'g') = 6.26 volt .

Example L.7.2 For the circuit shown in fig.7.3(a), the value of V_{s1} and I_s are fixed. When $V_{s2} = 0$, the current $I = 4A$. Find the value of I when $V_{s2} = 32V$.

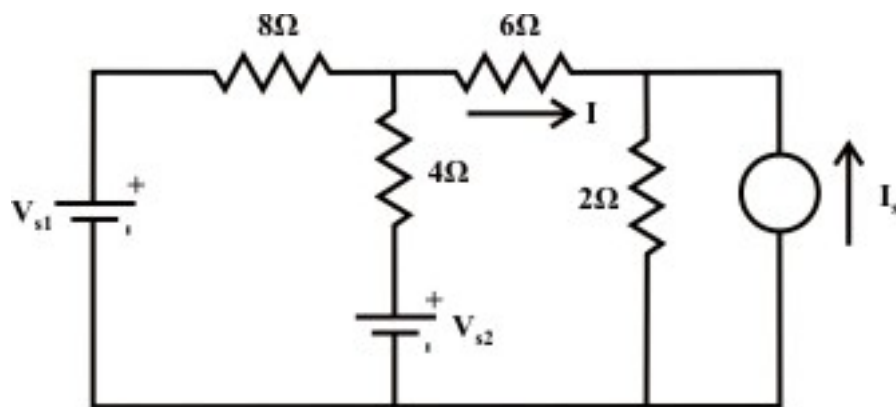


Fig. 7.3(a)

Solution: Let us assume that the current flowing 6Ω resistors due to the voltage and current sources are given by (assume circuit linearity)

$$I = \alpha V_{s1} + \beta V_{s2} + \eta I_s = I'_{(\text{due to } V_{s1})} + I''_{(\text{due to } V_{s2})} + I'''_{(\text{due to } I_s)} \quad (7.1)$$

where the parameters α , β , and η represent the positive constant numbers. The parameters α and β are the total conductance of the circuit when each voltage source acting alone in the circuit and the remaining sources are replaced by their internal resistances. On the other hand, the parameter η represents the total resistance of the circuit when the current source acting alone in the circuit and the remaining voltage sources are replaced by their internal resistances. The expression (7.1) for current I is basically written from the concept of superposition theorem.

From the first condition of the problem statement one can write an expression as (when the voltage source V_{s1} and the current source I_s acting jointly in the circuit and the other voltage source V_{s2} is not present in the circuit.)

$$4 = I = \alpha V_{s1} + \eta I_s = I'_{(\text{due to } V_{s1})} + I'''_{(\text{due to } I_s)} \quad (\text{Note both the sources are fixed}) \quad (7.2)$$

Let us assume the current following through the 6Ω resistor when all the sources acting in the circuit with $V_{s2} = 32V$ is given by the expression (7.1). Now, one can determine the current following through 6Ω resistor when the voltage source $V_{s2} = 32V$ acting alone in the circuit and the other sources are replaced by their internal resistances. For the circuit shown in fig.7.3 (b), the current delivered by the voltage source to the 6Ω resistor is given by

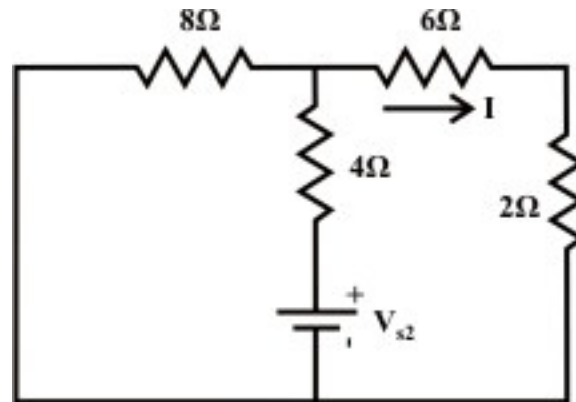


Fig. 7.3(b)

$$I_1 = \frac{V_{s2}}{R_{eq}} = \frac{32}{(8 \parallel 8) + 4} = 4 \text{ A} \quad (7.3)$$

The current following through the 6Ω due to the voltage source $V_{s2} = 32V$ only is 2 A (flowing from left to right; i.e. in the direction as indicated in the figure 7.3(b)). Using equation (7.1), the total current I flowing the 6Ω resistor can be obtained as

$$I = \alpha V_{s1} + \beta V_{s2} + \eta I_s = I'_{(\text{due to } V_{s1})} + I''_{(\text{due to } V_{s2})} + I'''_{(\text{due to } I_s)} = \left[I'_{(\text{due to } V_{s1})} + I'''_{(\text{due to } I_s)} \right] + I''_{(\text{due to } V_{s2})}$$

$$= 4 \text{ A} + 2 \text{ A} = 6 \text{ A} \quad (\text{note: } I'_{(\text{due to } V_{s1})} + I'''_{(\text{due to } I_s)} = 4 \text{ A} \text{ (see eq. 7.2)})$$

Example L.7.3: Calculate the current I_{ab} flowing through the resistor 3Ω as shown in fig.7.4(a), using the superposition theorem.

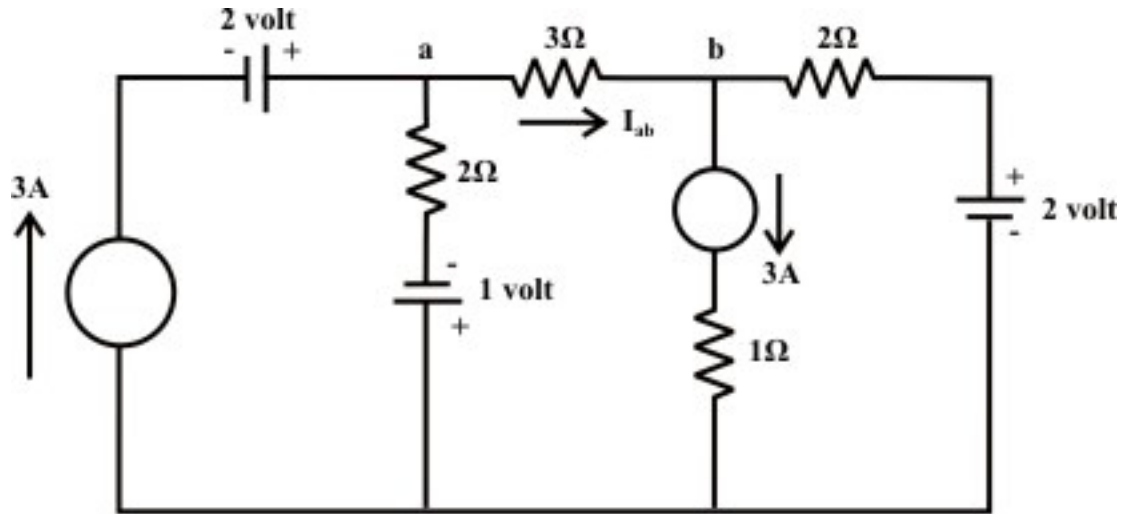


Fig. 7.4 (a)

Solution: Assume that the current source 3 A (left to the 1 volt source) is acting alone in the circuit and the internal resistances replace the other sources. The current flowing through 3Ω resistor can be obtained from fig.7.4(b)

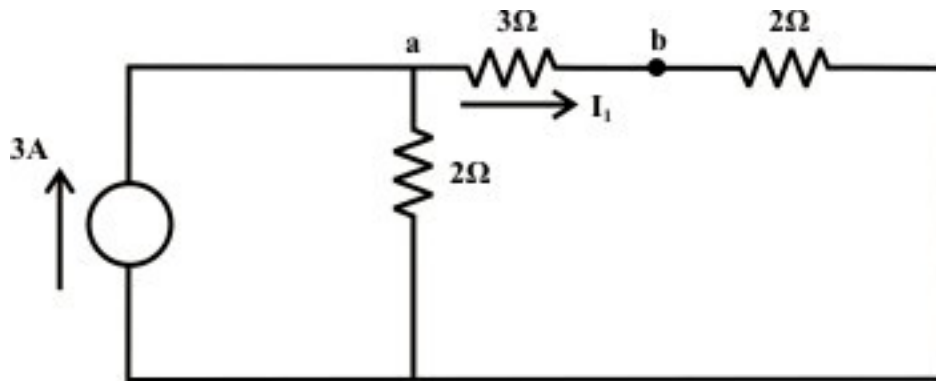


Fig. 7.4 (b)

and it is given by

$$I_{1(\text{due to } 3 \text{ A current source})} = 3 \times \frac{2}{7} = \frac{6}{7} \text{ A (a to b)} \quad (7.4)$$

Current flowing through 3Ω resistor **due to 2V source (only)** can be obtained from fig.7.4(c)

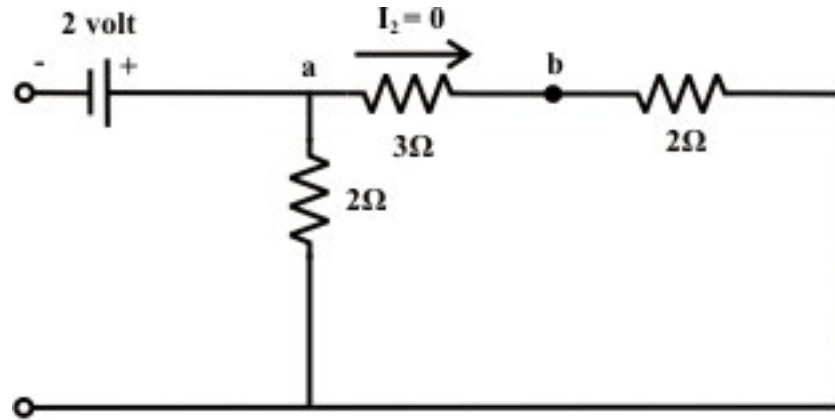


Fig. 7.4 (c)

and it is seen from no current is flowing.

$$I_{2(\text{due to } 2V \text{ voltage source})} = 0 \text{ A (a to b)} \quad (7.5)$$

Current through 3Ω resistor **due to 1V voltage source only** (see fig.7.3(d)) is given by

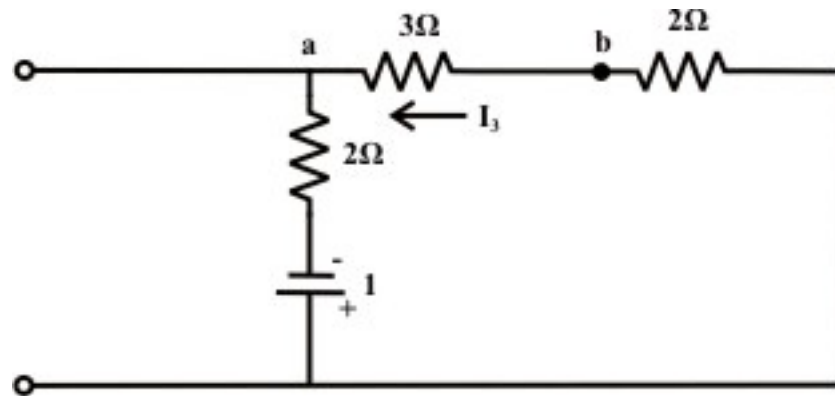


Fig. 7.4 (d)

$$I_{3(\text{due to } 1V \text{ voltage source})} = \frac{1}{7} \text{ A (b to a)} \quad (7.6)$$

Current through 3Ω resistor **due to 3A current source only** (see fig.7.3(e)) is obtained by

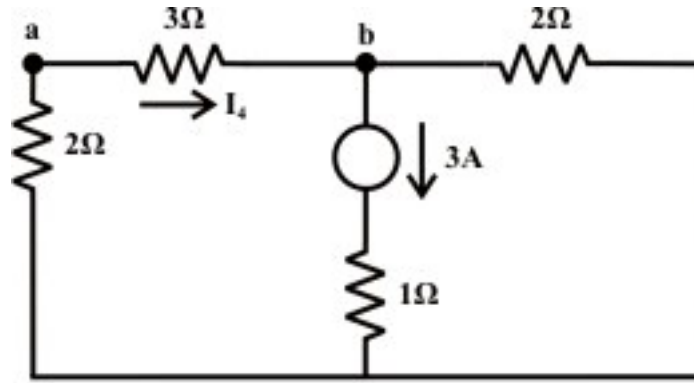


Fig. 7.4 (e)

$$I_{4(\text{due to } 3\text{A current source})} = 3 \times \frac{2}{7} = \frac{6}{7} \text{ A (a to b)} \quad (7.7)$$

Current through 3Ω resistor **due to 2V voltage source only** (see fig.7.3(f)) is given by

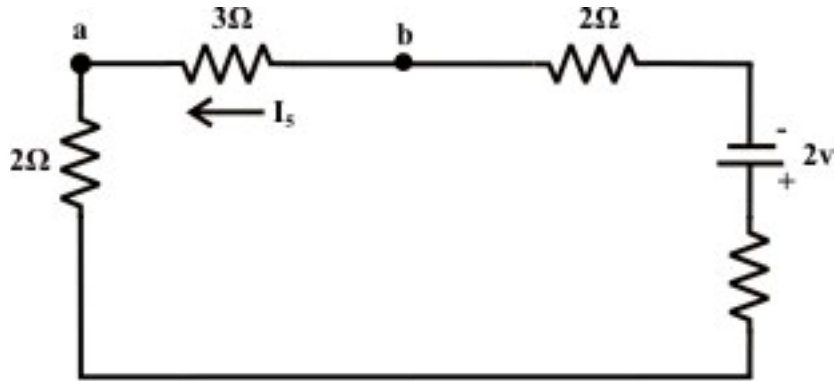


Fig. 7.4 (f)

$$I_{5(\text{due to } 2\text{V voltage source})} = \frac{2}{7} \text{ A (b to a)} \quad (7.8)$$

Resultant current I_{ab} flowing through 3Ω resistor due to the combination of all sources is obtained by the following expression (the algebraic sum of all currents obtained in eqs. (7.4)-(7.8) with proper direction of currents)

$$\begin{aligned} I_{ab} &= I_{1(\text{due to } 3\text{A current source})} + I_{2(\text{due to } 2\text{V voltage source})} + I_{3(\text{due to } 1\text{V voltage source})} + I_{4(\text{due to } 3\text{A current source})} \\ &\quad + I_{5(\text{due to } 2\text{V voltage source})} \\ &= \frac{6}{7} + 0 - \frac{1}{7} + \frac{6}{7} - \frac{2}{7} = \frac{9}{7} = 1.285 \text{ (a to b)} \end{aligned}$$

L.7.3 Limitations of superposition Theorem

- Superposition theorem doesn't work for power calculation. Because **power calculations** involve either the product of voltage and current, the square of current or the square of the voltage, they are not linear operations. This statement can be explained with a simple example as given below.

Example: Consider the circuit diagram as shown in fig.7.5.

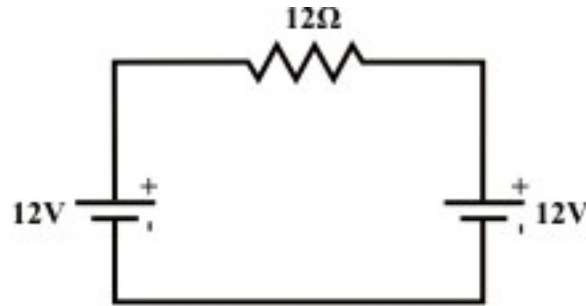


Fig. 7.5

Using superposition theorem one can find the resultant current flowing through 12Ω resistor is zero and consequently power consumed by the resistor is also zero. For power consumed in an any resistive element of a network can not be computed using superposition theorem. Note that the power consumed by each individual source is given by

$$P_{W1(\text{due to } 12V \text{ source}(\text{left}))} = 12 \text{ watts}; P_{W2(\text{due to } 12V \text{ source}(\text{right}))} = 12 \text{ watts}$$

The total power consumed by $12\Omega = 24 \text{ watts}$ (applying superposition theorem). This result is wrong conceptually. In fact, we may use the superposition theorem to find a current in any branch or a voltage across any branch, from which power is then can be calculated.

- Superposition theorem can not be applied for non linear circuit (Diodes or Transistors).
- This method has weaknesses:- In order to calculate load current I_L or the load voltage V_L for the several choices of load resistance R_L of the resistive network , one needs to solve for every source voltage and current, perhaps several times. With the simple circuit, this is fairly easy but in a large circuit this method becomes an painful experience.

L.7.4 Test Your Understanding

[Marks: 40]

T.7.1 When using the superposition theorem, to find the current produced independently by one voltage source, the other voltage source(s) must be ----- and the current source(s) must be -----.

[2]

T.7.2 For a linear circuit with independent sources $p_1, p_2, p_3, \dots, p_n$ and if y_i is the response of the circuit to source p_i , with all other independent sources set to zero), then resultant response $y = \dots$. [1]

T.7.3 Use superposition theorem to find the value of the voltage v_a in fig.7.6. [8]

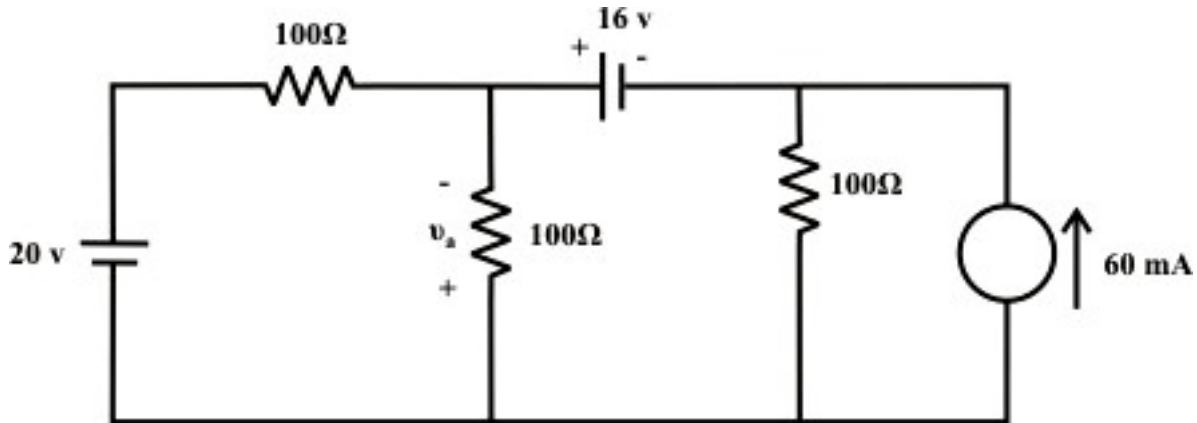


Fig. 7.6

(Ans. 14volts)

T.7.4 For the circuit shown in fig.7.7, calculate the value of source current I_x that yields $I = 0$ if V_A and V_C are kept fixed at 7V and 28V. [7]

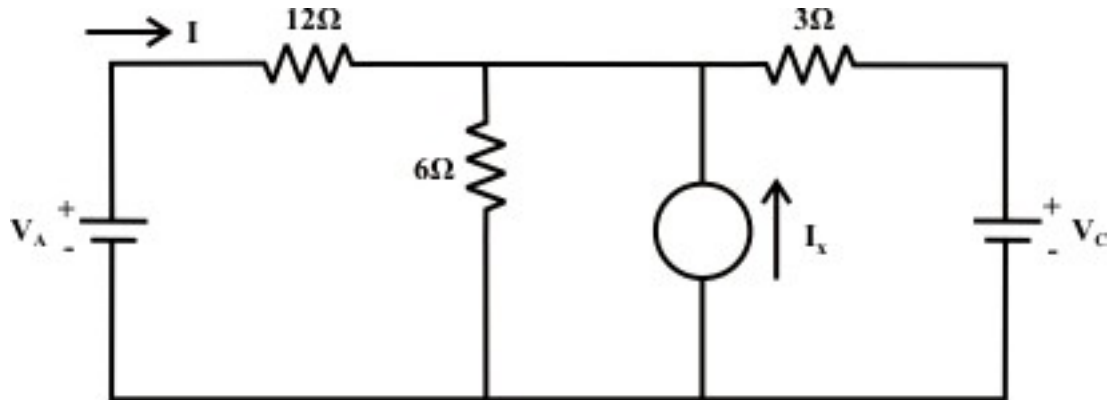


Fig. 7.7

(Ans. $I_x = -5.833 \text{ A}$)

T.7.5 For the circuit shown below (see fig.7.8), it follows from linearity that we can write $V_{ab} = \alpha I_x + \beta V_A + \eta V_B$, where α, β , and η are constants. Find the values of (i) α (ii) β and (iii) η . [7]

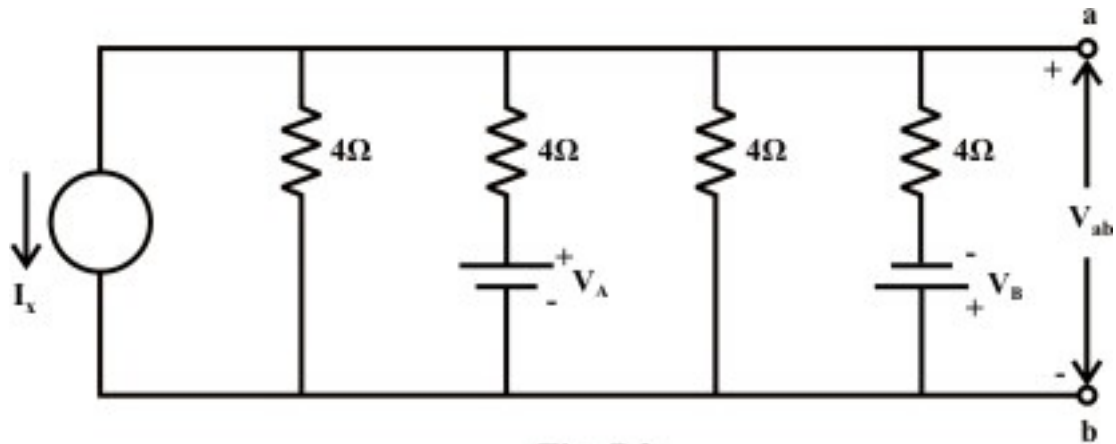


Fig. 7.8

(Ans. $\alpha = -1$; $\beta = 0.063$; and $\eta = -0.063$)

T.7.6 Using superposition theorem, find the current i through 5Ω resistor as shown in fig.7.9. [8]

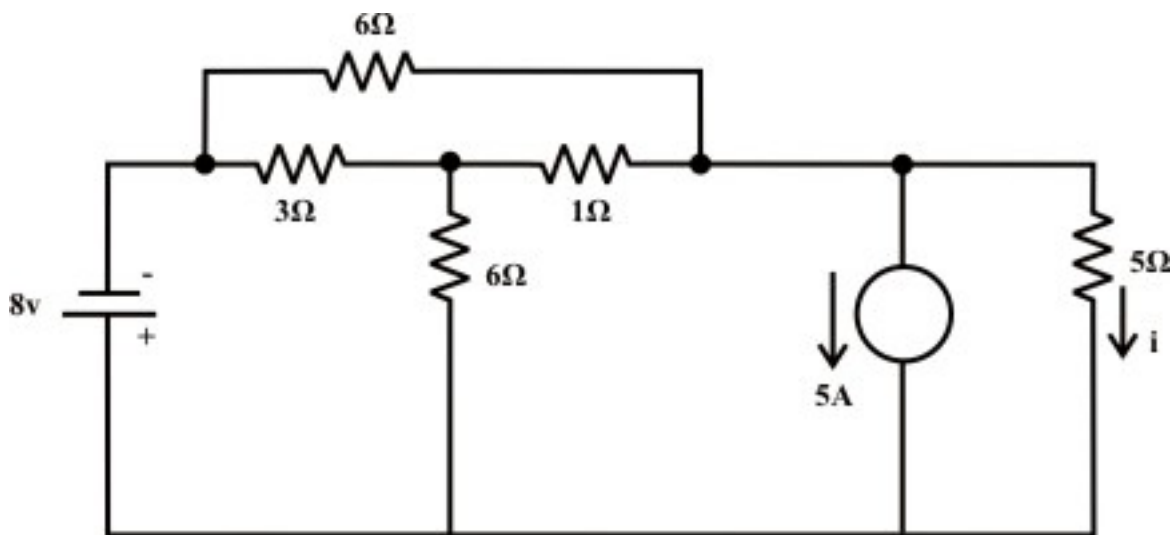


Fig. 7.9

(Ans. -0.538 A)

T.7.7 Consider the circuit of fig.7.10

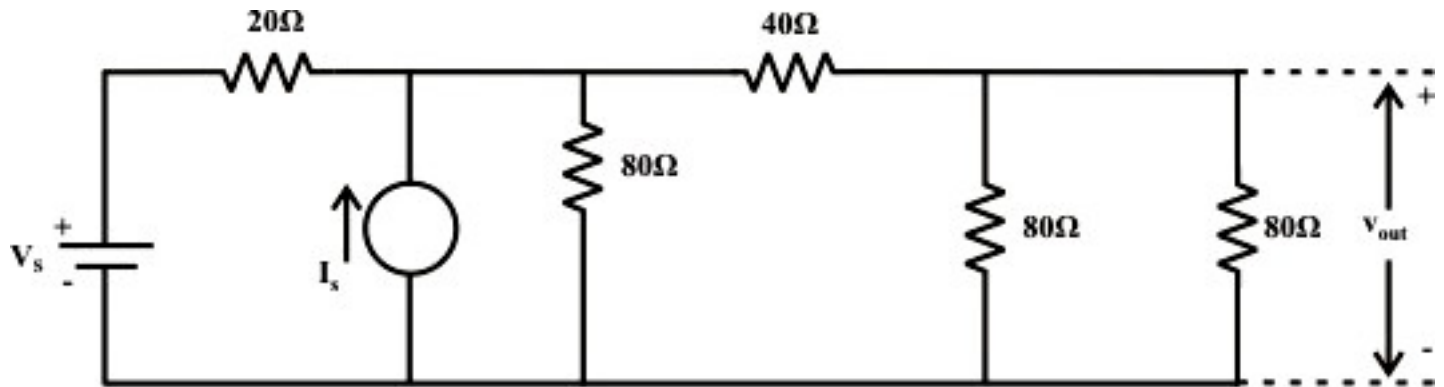


Fig. 7.10

- (a) Find the linear relationship between V_{out} and input sources V_s and I_s
 (b) If $V_s = 10V$ and $I_s = 1$, find V_{out}
 (c) What is the effect of doubling all resistance values on the coefficients of the linear relationship found in part (a)? [7]

(Ans. (a) $V_{out} = 0.3333V_s + 6.666I_s$; (b) $V_{out} = 9.999V$ (c) $V_{out} = 0.3333V_s + 13.332I_s$)